

# Higher Order Squeezing and Minimum Total Noise in Different Optical Processes

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## ABSTRACT

The possibility of observing higher order squeezing in different optical processes, such as four wave mixing, six wave mixing and second harmonic generation have been studied and it is shown that the higher order squeezing appears in all these cases. It is also shown that the minimum total noise ( $T_{\min}$ ) of a higher order squeezed state, which is a measure of the total fluctuations in the field amplitude, always increases with the increase in squeezing and thus we can use  $T_{\min}$  as an indirect measure of higher order squeezing.

## 1. Introduction

A nonclassical state of the electromagnetic field is one for which the Glauber-Sudarshan P-function [1] is either negative or more singular than delta function. Alternatively, if the P-function is not as well defined as the probability density is [2], then we obtain a nonclassical state. The nonclassical states do not have any classical analogue. Commonly, standard deviation of an observable is considered to be the most natural measure of quantum fluctuation [3] associated with that observable and the reduction of quantum fluctuation below the coherent state level corresponds to a nonclassical state. For example, an electromagnetic field is said to be electrically squeezed field if uncertainties in the quadrature phase observable  $X$  reduces below the coherent state level (i.e.  $(\Delta X)^2 < \frac{1}{2}$ ) and antibunching is defined as a phenomenon in which the fluctuations in photon number reduces below the Poisson level (i.e.  $(\Delta N)^2 < \langle N \rangle$ ) [4, 5]. Standard deviations can also be combined to form some complex measures of nonclassicality, which may increase with the increasing nonclassicality. As an example, we can note that the total noise of a quantum state which, is a measure of the total fluctuations of the amplitude, increases with the increasing nonclassicality in the system [6].

Probably, antibunching and squeezing are the most popular examples of nonclassical states and people have shown serious interest on these states since 1960s. But higher order extension of these nonclassical states are only introduced in late 1980s [7-10]. Among these higher order nonclassical effects, higher order squeezing has drawn the greater attention of the community [7, 8, 11, 12]. In the present work also we continue with the higher order squeezing. In next section we have briefly introduced the idea of higher order squeezing and total noise. In sections 3 and 4 we have reported that the generation of higher order squeezed state is possible by using six wave mixing and four

wave mixing processes respectively. In these sections analytic expressions of  $T_{\min}$  is also reported and it is shown that the degree of higher order squeezing can be measured through  $T_{\min}$ . We finish with some comments and concluding remarks in section 5.

## 2. Higher Order Squeezing and Total Noise

Higher order squeezing is defined in various ways. The definition which we have used in this work is by Hillery [11]. This definition is different from that of Hong and Mandel [7]. Present definition is also called amplitude squared squeezing. According to this definition of higher order squeezing, higher order quadrature variables are defined as

$$Y_1 = \frac{1}{\sqrt{2}}(a^{+2} + a^2) \quad (1)$$

and

$$Y_2 = \frac{i}{\sqrt{2}}(a^{+2} - a^2) \quad (2)$$

From the commutation relation  $[Y_1, Y_2] = i(4N + 2)$  it is easy to conclude that a state is squeezed in  $Y_1$  variable if

$$(\Delta Y_1)^2 < \langle 2N + 1 \rangle \quad (3)$$

or if,

$$f = (\Delta Y_1)^2 - \langle 2N + 1 \rangle < 0 \quad (4)$$

Total noise of a quantum state is a measure of the total fluctuations of the amplitude. For a single mode quantum state having density matrix  $\rho$  it is defined as

$$T(\rho) = \left( \Delta X \right)^2 + \left( \Delta \dot{X} \right)^2. \quad (5)$$

The total noise used to increase with the increase in nonclassicality in the system. This can be seen clearly if we associate total noise with higher squeezing. This can be done by following the prescription of Hillery [11]. In analogy to Hillery's work, we can write following uncertainty relations for the quadrature variables used in this work:

$$\left( \Delta X \right)^2 \left( \Delta Y_1 \right)^2 \geq \frac{1}{2} \left\langle \dot{X} \right\rangle^2 \quad (6)$$

and

$$\left( \Delta \dot{X} \right)^2 \left( \Delta Y_1 \right)^2 \geq \frac{1}{2} \left\langle X \right\rangle^2. \quad (7)$$

We can combine the above relations with the identity

$$2\langle N \rangle + 1 = \langle X \rangle^2 + \langle \dot{X} \rangle^2 \quad (8)$$

to obtain

$$\frac{1}{2} \left( 2\langle N \rangle + 1 \right) = \frac{1}{2} \left( \langle X \rangle^2 + \langle \dot{X} \rangle^2 \right) \leq \left( (\Delta X)^2 + (\Delta \dot{X})^2 \right) (\Delta Y_1)^2$$

or

$$T = (\Delta X)^2 + (\Delta \dot{X})^2 \geq \frac{1}{2} \frac{(2\langle N \rangle + 1)}{(\Delta Y_1)^2}. \quad (9)$$

From the condition (4) we know that for higher order squeezing  $(2\langle N \rangle + 1) > (\Delta Y_1)^2$ . Therefore,  $T$  will be greater than  $\frac{1}{2}$ . The more nonclassical the state is (the more negative  $f$  is) the more is the lower bound of the total noise

$$T_{\min} = \frac{1}{2} \frac{(2\langle N \rangle + 1)}{(\Delta Y_1)^2}. \quad (10)$$

Thus  $T_{\min}$  may be considered as a measure of depth of nonclassicality. In next two sections we will realize this with particular examples.

### 3. Six Wave Mixing Process

Six wave mixing may happen in different ways. One way is that two photon of frequency  $\omega_1$  are absorbed (as pump photon) and three photon of frequency  $\omega_2$  and another of frequency  $\omega_3$  are emitted. The Hamiltonian representing this particular six wave mixing process is

$$H = a^+ a \omega_1 + b^+ b \omega_2 + c^+ c \omega_3 + g(a^{+2} b^3 c + a^2 b^{+3} c^+) \quad (11)$$

where  $a$  and  $a^+$  are annihilation and creation operators in pump mode which satisfies  $[a, a^+] = 1$ , similarly  $b, b^+$  and  $c, c^+$  are annihilation and creation operators in stokes and signal mode respectively and  $g$  is the coupling constant.

Substituting  $A = a e^{i\omega_1 t}$ ,  $B = b e^{i\omega_2 t}$  and  $C = c e^{i\omega_3 t}$ , we can write the Hamiltonian in equation (11) as

$$H = A^+ A \omega_1 + B^+ B \omega_2 + C^+ C \omega_3 + g(A^{+2} B^3 C + A^2 B^{+3} C^+) \quad (12)$$

where  $\hbar = 1$ . A second order short-time operator solution of this Hamiltonian is already reported [13] as

$$\begin{aligned} A(t) = & A - 2igtA^+ B^3 C \\ + & g^2 t^2 [2AB^{+3} B^3 C^+ C - 9A^+ A^2 B^{+2} B^2 C^+ C - 18A^+ A^2 B^+ B C^+ C \\ - & A^+ A^2 B^{+3} B^3 - 9A^+ A^2 B^{+2} B^2 - 18A^+ A^2 B^+ B - 6A^+ A^2 C^+ C - 6A^+ A^2] \end{aligned} \quad (13)$$

or,

$$\begin{aligned} A(t) = & A - 2igtA^+ B^3 C \\ + & g^2 t^2 [2AB^{+3} B^3 N_C - 9N_A AB^{+2} B^2 N_C - 18N_A AN_B N_C \\ - & N_A AB^{+3} B^3 - 9N_A AB^{+2} B^2 - 18N_A AN_B - 6N_A AN_C - 6N_A A] \end{aligned} \quad (14)$$

where  $N_A = A^+ A$ ,  $N_B = B^+ B$  and  $N_C = C^+ C$ . This solution is valid for a short time. But the short time approximation technique is a very strong technique since this straight forward prescription is valid for any optical process where interaction time is short. Now we can use the analytic expression for time evolution of annihilation operator to check whether higher order squeezed state can be generated via six wave mixing process or not.

From equation (13 and 1) a straight forward but strenuous calculation yields

$$(\Delta Y_1)^2 = \langle Y_1^2 \rangle - \langle Y_1 \rangle^2 = 2|\alpha|^2 - 12g^2 t^2 [2|\alpha|^4 (3 + \cos(4\theta)) + 8|\alpha^2| + 1] \quad (15)$$

where  $\alpha = |\alpha| \exp(i\theta)$ . Here we would like to note that in the present study, we have taken all the expectation values with respect to the initial state  $|\alpha\rangle|0\rangle|0\rangle$  for simplification. This assumption physically means that initially a coherent state (say, a laser) is used as pump and before the interaction of the pump with atom, there was no photon in signal mode ( $b$ ) or stokes mode ( $c$ ). Here  $|\alpha\rangle$  is the coherent state which satisfies  $A|\alpha\rangle = \alpha|\alpha\rangle$ . By taking the expectation value of  $N(t) = a^+(t)a(t)$  we obtain

$$\langle N \rangle = |\alpha|^2 - 12g^2 t^2 |\alpha|^4. \quad (16)$$

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<sup>1</sup> A short time approximated expression of time evolution of annihilation operator in pump mode of six wave mixing process described by (11) is also derived in [12] but unfortunately their solution contain some mistakes.

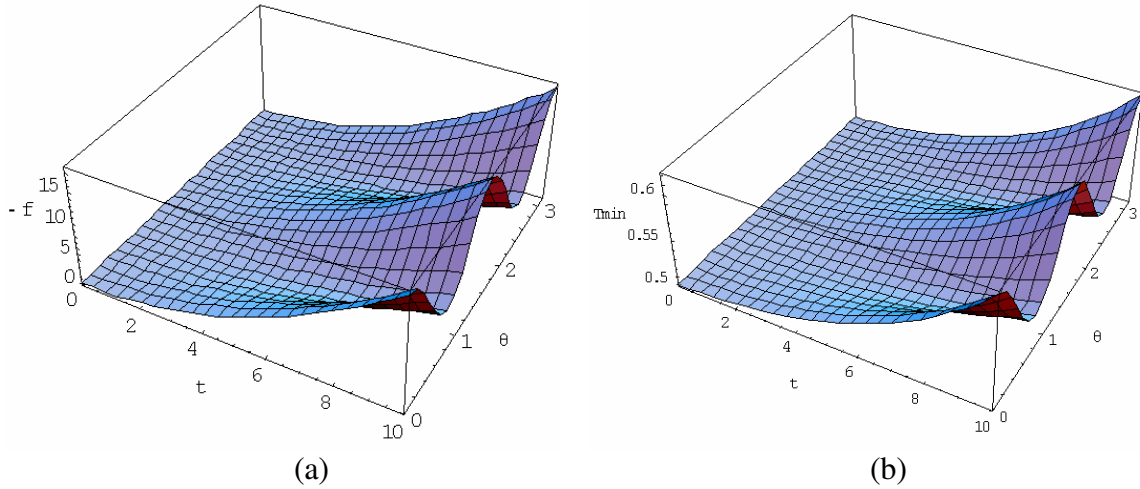
Now we can substitute (15) and (16) in (4) to obtain a closed form analytic expression for  $f$  as

$$f = (\Delta Y_1)^2 - \langle 2N + 1 \rangle = -12g^2t^2 \left[ 2|\alpha|^4 (2 + \cos(4\theta)) + 8|\alpha|^2 + 1 \right] \quad (17)$$

and similarly substituting (15) and (16) in (10) we have

$$T_{\min} = \frac{1}{2} \frac{(2\langle N \rangle + 1)}{(\Delta Y_1)^2} = \frac{1}{2} \frac{(2|\alpha|^2 + 1 - 24g^2t^2|\alpha|^4)}{2|\alpha|^2 + 1 - 12g^2t^2[2|\alpha|^4(3 + \cos(4\theta)) + 8|\alpha|^2 + 1]}. \quad (18)$$

From (17) we can observe that  $f$  is always negative, thus we always have higher order antibunching within the short time domain of the validity of the second order solution. The degree of higher order squeezing varies with the phase of the input coherent light  $\theta$ , initial photon number  $|\alpha|^2$  and the interaction time  $t$ . All these parameters can be tuned to control the depth of nonclassicality by increasing the negativity of  $f$ . From the Figure 1 below it will be clear how the depth of nonclassicality in higher order squeezing is reflected through the minimum total noise ( $T_{\min}$ ).



**Figure 1:** a) Variation of  $-f$  for six wave mixing process with respect to interaction time  $t$  and initial phase of the coherent state  $\theta$ . b) Variation of  $T_{\min}$  of six wave mixing process with respect to interaction time  $t$  and initial phase of the coherent state  $\theta$ . Here,  $g^2 = 10^{-6}$  and  $|\alpha|^2 = 50$ .

#### 4. Four Wave Mixing Process

Similarly, four wave mixing may also happen in different ways. One way is that in which two photon of frequency  $\omega_1$  are absorbed (as pump photon) and one photon of frequency  $\omega_2$  and another of frequency  $\omega_3$  are emitted. The Hamiltonian representing this particular four wave mixing process is

$$H = a^+ a \omega_1 + b^+ b \omega_2 + c^+ c \omega_3 + g(a^{+2} b c + a^2 b^+ c^+) \quad (19)$$

Short-time approximated second order solution of this Hamiltonian is [13]

$$A(t) = A - 2igtA^+BC + \frac{g^2 t^2}{2!} [4AB^+BC^+C - 2A^+A^2B^+B - 2A^+A^2C^+C - 2A^+A^2] \quad (20)$$

or

$$A(t) = A - 2igtA^+BC + g^2 t^2 [2AN_B N_C - N_A AN_B - N_A AN_C - N_A A] \quad (21)$$

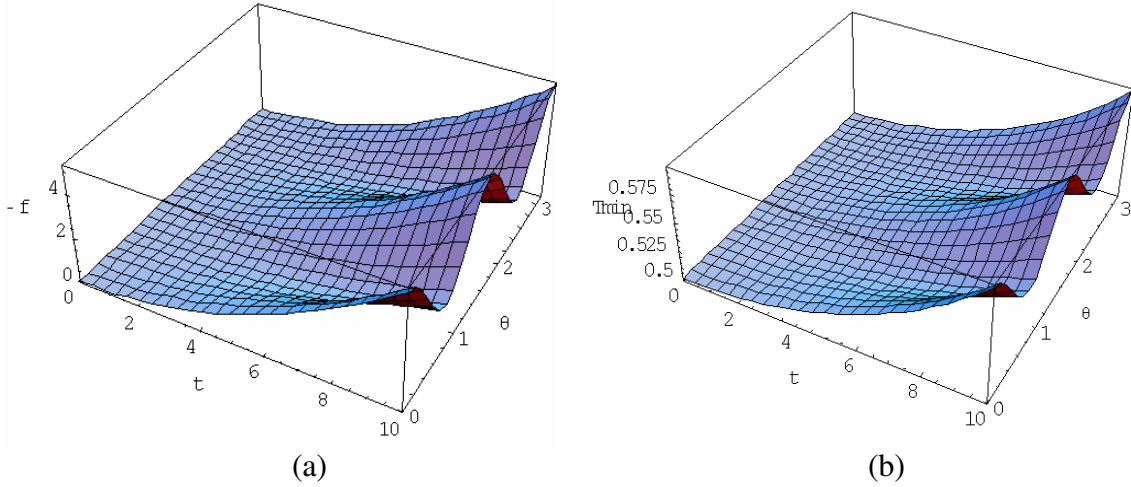
The respective values of  $f$  and  $T_{\min}$  can similarly be calculated as we have done for six wave mixing and that yields

$$f = -2g^2 t^2 [2|\alpha|^4 (2 + \cos(4\theta)) + 8|\alpha|^2 + 1] \quad (22)$$

and

$$T_{\min} = \frac{1}{2} \frac{(2|\alpha|^2 + 1 - 4g^2 t^2 |\alpha|^4)}{[2|\alpha|^2 + 1 - 2g^2 t^2 (2|\alpha|^4 (3 + \cos(4\theta))) + 8|\alpha|^2 + 1]} \quad (23)$$

Here  $f$  is negative and thus satisfy the criterion for higher order squeezing now from Figure 2 one can easily visualize that  $T_{\min}$  is a good measure of depth of nonclassicality of a higher order squeezed state.



**Figure 2:** a) Variation of  $-f$  for four wave mixing process with respect to interaction time  $t$  and initial phase of the coherent state  $\theta$ . b) Variation of  $T_{\min}$  of four wave mixing process with respect to interaction time  $t$  and initial phase of the coherent state  $\theta$ . Here,  $g^2 = 10^{-5}$  and  $|\alpha|^2 = 20$ .

## 5. Summary and Concluding Remarks

From equations (17) and (22), we can see that  $f$  is always negative within the domain of the validity of the solution. Therefore, higher order squeezed state can be generated via six wave mixing and four wave mixing processes. Again, from equations (4) and (9-10) it is clear that the total noise  $T_{\min}$  is a good measure of the depth of nonclassicality. This fact is more clearly visualized in figure 1 and 2. Here we also observe that  $f$  and  $T_{\min}$  depends on the initial phase of the coherent state ( $\theta$ ), number of photons present in the radiation field prior to the interaction ( $|\alpha|^2$ ) and the interaction time  $t$ . Since all these quantities can be tuned, one can also tune the depth of nonclassicality and the amount of total noise present in the system.

## References

- [1] Meystre P and Sargent M III, *Elements of Quantum Optics*, second edition Springer-Verlag, Berlin 1991.
- [2] Dodonov V V and Mon'ko V I (editors), *Theory of Nonclassical states of light*, Taylor & Francis, New York 2003.
- [3] Orłowski A, Phys. Rev. A **48** (1993) 727.
- [4] Dodonov V V, J. Opt. B. Quant. and Semiclass. Opt. 4 (2002) R1.
- [5] Hanbury-Brown R, Twiss R Q, Nature **177** 27 (1956).
- [6] Hillery M, Phys. Rev. A **39** 2994 (1989).
- [7] Hong C K and Mandel L, Phys. Rev. Lett. **54** 323 (1985).
- [8] Hong C K and Mandel L, Phys. Rev. A **32** (1985) 974.
- [9] Lee C T, Phys. Rev. A **41** 1721 (1990).
- [10] Lee C T, Phys. Rev. A **41** 1569 (1990).
- [11] Hillery M, Phys. Rev. A **36** 3796 (1987).
- [12] Giri D K and Gupta P S, Optics Communication **221** (2003) 135.
- [13] Gupta P, Pandey P N and Pathak A, quant-ph/0508060.